Introduction to Optimization

Jong-June Jeon

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Department of Statistics, University of Seoul

과목 소개

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- 교재: Convex optimization, Stephen Boyd

교수 소개

- Jong-June Jeon
- Office: 미래관 712
- Research areas: Synthetic Data Generation, GNN, Data Augmentation, Information Retrieval

Keywords

- Optimization problem
- Objective function, variable, constraints, feasible set, solution
- Unconstrained optimization problem, constrained optimization problem
- ...

What is the optimization problem? Decision-making under constraints.

Let $f : \mathbb{R}^p \mapsto \mathbb{R}$ and $X \subset \mathbb{R}^p$. The optimization problem is typically given by

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{st.} \quad & x \in X. \end{aligned}$$

Here we denote f as $f : x \mapsto f(x)$. We call x a variable, f an objective function, X a feasible set, and $x \in X$ the constraint.

Example 1 (Regression problem)

Let $(y_i, x_i) \in \mathbb{R} \times \mathbb{R}^p$ for $i = 1, \cdots, n$ be response-predictor pairs (constants) and $\beta \in \mathbb{R}^p$ be variable.

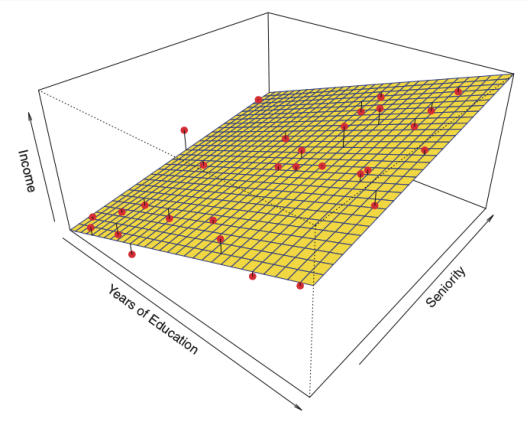


Figure 1: Regression surface

Example 1 (Regression problem)

$$L : \beta \mapsto \frac{1}{n} \sum_{i=1}^{n} (y_i - x_i^\top \beta)^2.$$

The solution of the following optimization problem is obtained by solving

$$\begin{aligned} \min_{\beta} \quad & L(\beta) \\ \text{st.} \quad & \beta \in \mathbb{R}^p. \end{aligned}$$

The solution is called the OLS (ordinary least squares) estimator. Since the feasible set is the whole set of $\mathbb{R}^p$, the optimization problem is called an unconstrained optimization problem.

Example 2 (Logistic regression problem)

Let $(y_i, x_i) \in \{0, 1\} \times \mathbb{R}^p$ be given response-predictor pairs and $\beta \in \mathbb{R}^p$.

$$L : \beta \mapsto -\frac{1}{n} \sum_{i=1}^{n} \left(y_i x_i^\top \beta - \log(1 + \exp(x_i^\top \beta)) \right)$$

The MLE of the logistic regression model is the solution of the following optimization problem:

$$\begin{aligned} \min_{\beta} \quad & L(\beta) \\ \text{st.} \quad & \beta \in \mathbb{R}^p. \end{aligned}$$

Example 3 (MLE for a general case)

Let $y|x \sim f(y|x;\theta)$ be the conditional density function of y for a given x, where $\theta \in \Theta \subset \mathbb{R}^p$ is the parameter of the density function. Let (y_i, x_i) be a random sample from $f(y|x;\theta)$. In our optimization problem, (y_i, x_i)s are constants, and θ is a variable.

$$L : \theta \mapsto -\sum_{i=1}^{n} \log f(y_i|x_i;\theta)$$

The MLE of the unknown θ is given by

$$\begin{aligned} \min_{\theta} \quad & L(\theta) \\ \text{st.} \quad & \theta \in \Theta. \end{aligned}$$

Example 4 (L_1-Compressed Sensing)

Let $Y \in \mathbb{R}^m$ be an original signal vector and $X \in \mathbb{R}^{m \times n}$ be a filter matrix and $\beta \in \mathbb{R}^n$ the sparse signal.

$$L : \beta \mapsto \|\beta\|_1$$

The L_1-Compressed Sensing is the solution to the following problem:

$$\begin{aligned} \min_{\beta} \quad & L(\beta) \\ \text{st.} \quad & \beta \in \{\beta : Y = X\beta\} \end{aligned}$$

Example 5 (Other machine learning models: Empirical Risk Minimization (ERM) Framework)

- Spatial Regression problem
- Nonnegative matrix factorization
- Learning a Deep Neural Network model
- A lot of problems we can solve!

Why did I study optimization methods?

- Naive approach: Let f be our objective function and $x \in X$ be constraint. Then investigate $f(x)$ for each $x \in X$ and choose the minimum value of $f(x)$.
- Unfortunately, this method is intractable in practice. Consider $x \in \mathbb{R}^{20}$.

Is it tough to learn and practice?

- If $X = \mathbb{R}^p$ (no constraint), it is surprisingly easy.
- If $X \subsetneq \mathbb{R}^p$, it depends on the problem.

We will deal with

(1) Unconstrained convex optimization problem.

(2) Linear constrained convex optimization problem.

(3) To solve (2) easier, we will learn general optimization methods.

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Let's start our journey to OPTIMIZATION!